# An ARITHMETICAL TREATISE OF THE Combinations, Permutations, Elections, and Composition, OF QUANTITIES.

ILLUSTRATED

By several Examples, with a New Speculation of the Differences of the Powers

OF

# NUMBERS.

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[illegible]

Cambridge

[illegible]

Printed for [illegible] the Ship in St. Paul's [illegible] 1759.

# TO THE READER.

*Courteous Reader,*

ABove a Year ſince, on the Entreaty of a very Worthy and Publick-Spirited Friend, I gave my Conſent that theſe Papers ſhould come to light; afterward underſtanding that ſome French Authors had writ on this Subject, I put a ſtop to the Preſs; at length having obtained thoſe Authors, and peruſed what they have ſaid concerning the Argument herein handled, was willing (to prevent any abuſes) that the Preſs ſhould proceed. I have ſince out of thoſe Books added two things; the firſt out of Malbranch, alias the Author of the Elemens des Mathematiques; namely, to give the number of the ſeveral Compoſitions that may be made of 24 Letters, and that on a double account, one to correct a Miſtake of the Printer of that Treatiſe; for I do ſuppoſe it to be no other, (for that the Number of Figures, as alſo the firſt and the laſt are

*true*;) *the other, to ſhew the manner of Operation, which he hath omitted. The Second is a Demonſtration out of Monſieur* Paſcal's *Tract* du Triangle Arithmetique *of what I had before by chance found out. Which you will find in Page* 33, *but miſplaced; for it ſhould follow Page* 42.

*In one place I have made uſe of* Logarithms *to calculate Numbers conſiſting of* 30 *Figures, which poſſibly may ſeem ſtrange to ſome, for that the Table of* Logarithms *which I uſed, doth not promiſe to give* Logarithms *beyond* 5 *places: To which Doubt I anſwer, that exactly they cannot work beyond* 5 *places, but if they conſiſt of* 30 *or more Figures, the firſt* 4 *Figures with the Number of Places will be true (as may eaſily be demonſtrated) which is enough to make good my Propoſition.*

Farewel.

## *Of the Several Combinations, Elections, Variations and Compositions of Quantities.*

1. BY Combination of Quantities, I mean how many ſeveral ways I may take any Number of Quantities out of any given Number of Quantities, without having any reſpect to their places; as how often out of the Engliſh Alphabet, I may take 2, 3, or more Letters; as two Letters, *a b*, *a c*, *c d*, *f g*, *f n*, &c. or three Letters, *a b c*, *c f l*, *b m g*, *h n p*, *c d e*.

2. By Election of Quantities, I mean (as *Francis Schoten* in his *Miſcellanies*) any Number of Quantities being given, how many ſeveral ways I may take them without having reſpect to their places; as, *a*, *b*, *c*, may be taken ſeven ways; as, *a*, *b*, *c*, *a b*, *a c*, *b c*, and *a b c*.

3. By Variations, permutation or changes of the Places of Quantities, I mean, how many ſeveral ways any given Number of Quantities may be changed, as in relation to their places: as, *a*, *b*, may be changed into *b*, *a*; *a*, *b*, *c*, ſix ways; *a b c*, *a c b*, *b a c*, *b c a*, *c a b*, and *c b a*.

4. By Compoſition of Quantities, which is the moſt compoſed way, I mean, when in any Number of given Quantities, as in Letters or Figures, one Row is joyned with another Row of the ſame, or with 2, 3 or more other Rows; as is to be ſeen in the Tables of Natural Numbers, the placing of Letters, and the chances of the Dice. This differs from Combination and

Election of Quantities; for that there one Quantity is taken but once, here, as often as there are Quantities to be taken.

## *Of the Combination of Quantities.*

5. As out of Ten Letters, *a*, *b*, *c*, *d*, *e*, *f*, *g*, *h*, *i*, *k*, or the Ten Figures, how many Combinations are there of Two Letters or Figures?

There is with *a*, nine Combinations, *viz.* *ab*, *ac*, *ad*, *ae*, *af*, *ag*, *ah*, *ai*, and *ak*: with *b*, 8 Combinations, beſides *ba*, already accounted, *viz.* *bc*, *bd*, *be*, *bf*, *bg*, *bh*, *bi*, *bk*; with *c*, 7 Combinations, beſides *ac*, *bc*; with *d*, 6 Combinations; with *e*, 5; with *f*, 4; with *g*, 3; with *h*, 2; with *i*, 1.

Their Number by the Table of Figurate Numbers, may eaſily be found thus;

Take the number of Letters uſed in the Combination, leſſened by an Unit, out of the Number given, and with that enter the ſecond Row of the Table of Figurate Numbers, and even with it in the Row belonging to the Combination, is the Number ſought; as 10—1=9; with it enter the Table, and in the third Row (there being but 2 Letters in the Combination) is 45, and ſo many ſeveral Combinations there are of 2 Letters in 10.

6. How many ſeveral Combinations are there of 3 Letters in ten? You muſt take 2 out of (the Number given) Ten, and with the remainder 8, enter the ſecond Row of the Table, and the Num. in the 4th. Row even with it, is 120; and ſo many ſeveral Combinations there are of 3 Letters in 10. The reaſon why 2 is taken from the given Number, is, for that the two firſt Letters do not vary the acceptation; as, *abc*, *abd*, *abe*, *abf*, *abg*, *abh*, *abi*, and *abk*. So that here are but eight ſeveral Combinations with *ab*. On the ſame account, if there be taken four Letters, there will be ſeven diverſe Combinations with *ab*; if five Letters, ſix Combinations with *a*, *b*, *c*, *d*.

7. How

7. How many ſeveral Combinations are there of four Quantities in Ten? 10—3=7; with which enter the ſecond Row of the Table, and even with it in the fifth Row, is 210; and ſo many ſeveral Combinations there are of Four Quantities in Ten; and proceeding after the ſame method, you will find there are 210 Combinations of ſix Quantities, 120 of ſeven Quantities, 252 Combinations of five Quantities, 45 of eight Quantities, 10 of nine Quantities, 1 of ten in ten Quantities.

8. If it be required to find how many ſeveral Combinations there are of two Letters in the Engliſh Alphabet; I ſay, 24—1=23; with which I enter the ſecond Row of the Table, and even with it in the third Row ſtands 276, and ſo many Combinations there are of 2 Letters in the Alphabet. If more Combinations are ſought after, as of 3, 4, 5, 6, or 7 Letters; then 24—2=22; even with it in the fourth Row ſtands 2024, the number of Combinations of 3 Letters in the Alphabet; over againſt 21 in the fifth Row, you will find 10626, the number of Combinations of 4 Letters; after the ſame manner, you will find 42504 Combinations of five Letters; 134596 of 6 Letters; 346104, of 7 Letters; 735461 of 8 Letters in the Alphabet.

9. But in caſe of Combinations, where there is no figurate Table, or the number of Quantities do exceed the Table; then thus.

Firſt, Place the number of Quantities given, then decreaſe that number by an Unite, as often as is the number of Quantities in the Combination, which place following one the other, with the Sign of Multiplication, ×, between them as the Dividend, and then place an Unite with the like number of Figures increaſing by an Unite, with the Sign of Multiplication ×, between them as a Diviſor.

Then (inſtead of multiplying thoſe Numbers according to their Signs for a Dividend and a Diviſor, as *Tacquet* teaches) prepare the Terms, by dividing the Diviſor and Dividend by

each ſeveral number in the Diviſor, then the Diviſor will be brought to an Unite; then multiply the remainder of your Dividend, and you have your deſire; as in the Example given *Sect.* 5, 6, 7. Where it is deſired to know how many Combinations there are of 2, 3, 4, 5, 6, 7, 8, 9, and 10 Quantities in Ten.

$$\begin{array}{l} 2\times1 \\ 1 \end{array} \Big) \begin{array}{l} 10\times9\ (45 \\ 5 \end{array}$$

Place 10 and 9 as the Dividend, 1 and 2 as the Diviſor, with the Sign of Multiplication × between them; then divide 2 in the Diviſor, and 10 in the Dividend by 2, the quotients are 1 and 5; which multiplied in 9, the Product 45 is the number of Combinations of 2 quantities in 10.

If you would know how many Combinations there are of three Quantities in ten, add 8 to the Dividend, and 3 to the Diviſor, with the Sign of Multiplication ×; thus,

$$\begin{array}{l} 3\times2\times1 \\ 1\times1 \end{array} \Big) \begin{array}{l} 10\times9\times8\ (5\times3\times8=120. \\ 5\times3 \end{array}$$

Afterwards prepare the Terms by dividing 9 in the Dividend, and 3 in the Diviſor, by 3, then your Quotient is $5\times3\times8=120$, the number of Combinations of three Quantities in ten. If you deſire to know the number of Combinations of four Quantities in ten, then add 7 to the Dividend, and 1 to the Diviſor,

$$\begin{array}{l} 4\times3\times2\times1 \\ 1\times1\times1 \end{array} \Big) \begin{array}{l} 10\times9\times8\times7(5\times3\times2\times7=210 \\ 5\times3\times2 \end{array}$$

and divide 4 in the Diviſor, and 8 in the Dividend, by 4, the Quotes are 1 and 2: then $5\times3\times2\times7=210$ the Combinations of four Quantities in ten.

If more Combinations are ſought, proceed as before;

$$\begin{array}{l} 9\times8\times7\times6\times5\times4\times3\times2\times1 \\ 1\times1\times1\times1\times1\times1\times1\times1 \end{array} \Big) \begin{array}{l} 10\times9\times8\times7\times6\times5\times4\times3\times2( \\ 5\times3\times2 \end{array}$$

The

| *The Factors of the Quotes.* | | *The Quotes.* | | *Letters.* |
|---|---|---|---|---|
| 5*9 | | 45 | | 2 |
| 5*3*8 | | 120 | | 3 |
| 5*3*2*7 | | 210 | | 4 |
| 3*2*7*6 | = | 252 | of | 5 in Ten. |
| 3*2*7*5 | | 210 | | 6 |
| 3*2*5*4 | | 120 | | 7 |
| 3*5*3 | | 45 | | 8 |
| 5*2 | | 10 | | 9 |

The Demonſtration hereof I defer to the laſt Sheet.

*For the finding ſeveral Combinations in the Engliſh Alphabet; thus.*

Dividend, 24*23*22*21*20*19*18*17.

Diviſor, 8*7*6*5*4*3*2*1.

| *The Factors of the Quotient.* | | *The Quot.* | | *Letters.* |
|---|---|---|---|---|
| 12*23 | | 276 | | 2 |
| 4*23*22 | | 2024 | | 3 |
| 23*22*21 | | 10626 | | 4 in the |
| 23*22*21*4 | = | 42504 | of | 5 Alpha- |
| 23*22*7*2*19 | | 134596 | | 6 bet. |
| 23*22*2*19*16 | | 346104 | | 7 |
| 23*11*19*9*17 | | 735471 | | 8 |

Where there are ſeveral Combinations ſought, it is beſt not to multiply the Factors that are in the Quotient together, until they are all found; for there are ſome Factors, which are common to moſt of the Quotients; as here 23*22=506 is common to the 5th middle Quotient.

10. It is deſired to know how many ſeveral Lifts there are of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 Cards in a Pack.

For as much as 52, the number of Cards in a Pack, do exceed my Table of Figurate Num. it may be reſolved this way.

12*11*10*9*8*7*6*5*4*3*2*1 ) 52*51*50*49*48*47 *46*45*44*43*42*41. (

The

| *The Factors of the Quotient.* | | *The Quotients.* | | *The several Lifts* | |
|---|---|---|---|---|---|
| 2, 26×51 | | 1326 | | 2 | |
| 3, 26×17×50 | | 22100 | | 3 | |
| 4, 26×17×25×49 | | 270725 | | 4 | |
| 5, 13×17×5×49×48 | | 2598960 | | 5 | *Cards in a Pack.* |
| 6, 13×17×5×49×8×47 | = | 20358520 | | 6 | |
| 7, 13×17×5×7×8×47×46 | | 133784560 | of | 7 | |
| 8, 13×17×5×7×47×46×45 | | 752438150 | | 8 | |
| 9, 13×17×5×7×47×46×5×44 | | 3679075400 | | 9 | |
| 10, 13×17×5×7×47×46×22×43 | | 15820024220 | | 10 | |
| 11, 13×17×5×7×47×46×2×43×42 | | 60403728840 | | 11 | |
| 12, 13×17×5×7×47×46×43×7×41 | | 206379406870 | | 12 | |

Here $13 \times 17 = 221$ is common to every *Quotient*.

For if you multiply it in 6, you have the first Quotient 1326, if in 100, the second, 22100.

Again, $13 \times 17 \times 5 \times 7 \times 47$ is common to the seven last Quotients.

So that if 1000 men shall constantly deal 12 Cards for 12 hours each day, excepting *Sundays*, they cannot deal all the several Games that are on 12 Cards in a Pack, in 54 Years, accounting that each man may deal a thousand Games in an hour.

11. How many several Stocks may there be at *Gleek*?

For asmmuch as *Gleek* is played but with 44 Cards, and there are 7 Cards in the Stock, besides the Card turned up *Trump*, we must enquire how many Combinations there are of 7 in 44.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 ) 44 \times 43 \times 42 \times 41 \times 40 \times 39 \times 38 ( 44 \times 13 \times 41 \times 13 \times 38.$

$= 38320568$, the Combinations of 7 Quantities in 44, which multiplied in 37; for that each Stock may have either of the remaining Cards, which are 37, turned up *Trump*, produces 1417861016, the number of the Several Stocks that may be at *Gleek*.

12. An old Captain having spent most part of his Time and Estate in the Wars, at the end thereof, desired of the King whom he served, as a recompence of his Service, a Farthing for

for each ſeveral File, containing ſix Souldiers, that he could make in his Company, which conſiſted of 100 Souldiers: The King conſidering his Requeſt, and the greatneſs of his Deſert, granted it. How much did it come to? *Anſw.* 1241721 *l.* 5 *s.*

$6*5*4*3*2*1)100*99*98*97*96*95\left(\begin{matrix}10*33*98*97*4*95\\=1192052400;\end{matrix}\right.$

or, $100*33*49*97*4*19=1192052400$, the number of Files or ſeveral Combinations of 6 in 100 Quantities, which being divided by 960, the Farthings in a Pound; the Quotient is 1241721, 25. *l.*

13. *Sempronius* bought of *Caius* 50 Sheep, to be drawn forth of 100, and being ſomewhat tedious in chooſing of them, *Caius* ſaid to *Sempronius*, give me one Barley-Corn for each ſeveral parcel of 50 Sheep that may be drawn forth of this 100, and you ſhall have the whole 100. I will, anſwered *Sempronius*; and gave him 5 *l.* in Earneſt: How much doth the value of the Sheep come to?

*Anſw.* That if the Terreſtrial Globe ſhould be covered with *Guineys* ten foot thick, there would not be enough to pay for the Sheep: Neither if the Terreſtrial Globe ſhould be converted to Barley, would there be Barley enough to ſatisfie for the Sheep. Which I thus demonſtrate,

Proceeding according to the foregoing Rules, you will find the Factors in the Quotients to be,

97*19*93*13*89*87*17*83*9*79*11*73*71*67*61*59*53*8; which if you work with Logarithms, (which will be exact enough) will amount unto 10088 with 25 Cyphers, whoſe Logar. is, 29, 003862 for the Barley-Corns to be paid for the Sheep, I have exactly weighed an Ounce *Aver-du-pois* of Barley, and it contained 681 Grains; and therefore one Pound contains 10896 Grains, and a Buſhel 544800 Gr. For I found that a Buſhel of the ſame Barley containing eight Gallons, did weigh 50 *l. aver-du-pois*, whoſe Logar. 5, 736236 being

being deducted from 29.003862. remains 23,167416, whose Number is 1851, with 20 Cyphers, for the Bushels of Barley to be paid for the Sheep.

Which accounting at 2 *s.* the Bushel, comes to 1851, with 19 Cyphers, for the worth of the Barley in Pounds.

The Circuit of the Earth, by the most common account, is, 21600 Miles; its Logarithm, 4,334452.

| | | |
|---|---|---|
| Which squared, is, | 466560000 | 8,668908. |
| To it add | | 1,502850. |
| Then you will have | 148411000. | 8,171758. |

The Superficies of the Earth in square Miles.

| | | |
|---|---|---|
| Every Mile is in length 5280 Feet. | Logar. | 3,722634 |
| Which squared, is | 27878400 | 7,445264 |
| Which added to the Log. of the square Miles, | | 8,171758 |
| renders 414021, with ten Cyphers | | 15,617022 |

Equal to the Number of square Feet, on the Surface of the Globe.

| | | |
|---|---|---|
| The Cubical inches in a Cubick Foot, | 1728. Log. | 3,237544 |
| Multiplied in the Ounces that are in a Cubical Inch of Gold, (*Mathematical Compendium*, p. 17.) | 9,91735. | 0,996396 |
| | | 4,233940 |

You have the Log. of the Ounces contained in a Cubical Foot of Gold; which divided by the Ounces in a ℔. 12. is 1,079181

| | | |
|---|---|---|
| You have the *Troy* Pounds in a Cubick Foot of Gold | 1728. | 3,154759 |
| Which multiplied by the value of 1 ℔. *Troy* of Gold, | 40,918. | 1,611913 |
| You have the value of a Cubick Foot of Gold. | 58435, 1. | 4,766672 |
| Which multiplied in the number of Feet, with ten Cyphers. | 414025, 15. 617022 | |
| | | 20,383694. |

You

You have 2419, with 17 Cyphers, for the value of the Gold that covers the Earth one Foot. 20,383694

Which deducted from the Log. of the value of the Barley } 22,267426 / 20,383694

Whose Number 74,77 1,873732

shews, That if the whole Surface of the Earth was covered with Gold 74 feet thick, it was but the price of the Barley. *q. e. d.*

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Again, to the Cube of the Circuit of the Earth, 100776960000000 13,003362

Add 2,227549

You have the solid Content of the Globe in Cubick Miles. } 1701820000000 11,230911

Which multiplied into the Cube of 5280, makes the Cubick Feet in a Cubick Mile, } 147197952000 11,167902

You have 2505 with 19 Cyphers, = to the Cubick Feet contained in the Globe. 22,398813

According to Mr. *Gunter* and Mr. *Oughtred*'s Experiments, 231 cubick inches are in a Gallon; therefore 1848 cubick inches are in 8 Gallons or a Bushel.

As 1848,3,266702, The Cubical inches in a Bushel.

is to 1728,?,237544, The Cubical Inches in a Cubical Foot.

So is 544800.5,736136, The Grains in a Bushel by trial,

to 509422.5,707078. The Grains in a Cubical Foot, Multiplied in 22,398813. The Cubical Feet in the Globe 1276

×25 cyphers, 28,105891, shews how many Barley-Corns if the

 Globe

Globe was converted to Barley, it would contain; which ſubſtracted from the Log. 29,003862, the price of the Sheep. 28,105891

7,906. 0,897971, ſhews that above 7 ſuch Worlds, if converted to Barley, is not enough to pay for the Sheep: *q. e. d.*

Or if the whole Globe was covered 3436 Miles, equal to the Semidiameter of the Earth high in Barley, there would not be enough.

For that would be equal to 7 times this Globe, whoſe Semidiameter is 3436 Miles; Spheres being in triplicate proportion of their Diameters 12 *E.* 8. that is here where the Diameters are as, 1 and 2, the Spheres will be as 1 and 8, take the difference of the Spheres, it will be as 1 to 7.

## *Of Election of Quantities.*

13. The Election of Quantities may likewiſe be found out by the Table of Figurate Numbers, or elſe by the foregoing Rule; for it is but the addition of the ſeveral Combinations that are under it to the Number it ſelf.

As, How many are the Elections of ten Quantities?

By the Table I muſt ſee how many Combinations there are of 2, 3, 4, 5, 6, 7, 8, 9, and 10, in ten Quantities, as is already done in the 5th, 6th, 7th, and 9th. Section, which added to 10, makes 1023, the Elections, thus;

10+45+120+210+252+210+120+45+10+1= 1023, the Elections of 10 Quantities.

14. How many Conjunctions may there be of the ſeven Planets?

The Election of 7 Quantities leſſened by 7, the number of Planets is equal to the Conjunction of the Planets; as, 2+135+35+21+7+1=120, the Conjunctions, that is to ſay, there are 21 Conjunctions of 2 Planets, 35 Conjunctions of 3 Planets,

35 of 4 Planets, 21 of 5 Planets, 7 of 6 Planets, and 1 of 7 Planets.

15. There is a Key that hath 8 ſeveral Wards, and it is deſired to know, how many Locks whoſe Wards differ, may be made which that Key may unlock? I anſwer, 255 Locks.

1 2 3 4 5 6 7 8

8+28+56+70+56+28+8+1=255 ſeveral Locks; 8 whereof, have but one ſingle Ward, 28 have double Wards, 56 treble Wards, 70, 4 Wards, 56, 5 Wards, 28, 6 Wards, 8, 7 Wards, and 1 Lock hath 8 Wards.

But if it had been required to know how many ſeveral Locks might have been made, which that Key might unlock, and neither of the other Keys? I Anſwer, 70 Locks; and each Lock muſt have 4 ſeveral Wards.

16. But in caſe of Elections, where it is deſired to have it performed at one Operation;

Then it may be done by the Rules of Geometrical Progreſſion.

Let $r$ = to the *Ratio*, which here is always 2.

$s$ = the Sum of the *Terms*.

$t$ = the number of the *Terms*. Then

$r^{(t)} - 1 = s$. That is to ſay, Multiply the *Ratio* 2, as often in it ſelf, as is the Number of *Terms*, and from it ſubſtract 1. the Remainer is the Sum of the Elections.

As the 10*th* power of 2 is 1024. from which ſubſtract 1. the Remainer ſhews its Elections.

17. How many Elections are there of the Letter of the Alphabet? $r(t) - 1$. *i. e.* $r^{(24)} - 1$.

$r^{(24)} = 16777216$ = to the 24*th* power of 2.

Subſtract 1.

16777215. The Number of Elections of 24 Letters.

That Elections do proceed as the Sums in Geometrical Progression, whose *Ratio* is 2. appears; as *Francis Schoten* in his *Miscellanies* hath demonstrated: as take 4 several Magnitudes, *a, b, c, d,* in order.

| *Sum* | *Elect.* | *Dif.* |
|---|---|---|
| *a.* | 1 | 1. |
| *a, b. ab.* | 3 | 2. |
| *a, b, c, ab, ac, bc, abc.* | 7 | 4. |
| *a, b, c, d. ab, ac, ad, bc, bd, cd, abd, abc, acd, bcd, abcd.* | 15 | 8. |

Or thus, leaving out those that are mentioned in the foregoing Election.

| | |
|---|---|
| *a.* | 1. |
| *b, ab.* | 2. |
| *c, ac, bc, abc.* | 4. |
| *d, ad, bd, abd, cd, acd, bcd, abcd.* | 8. |

So that these which are the Differences of the Elections, do all proceed in Geometrical Progression, and the Elections are the Sums of Quantities proceeding in Geometrical Progression.

| | Elect. | Dif. of Elections. |
|---|---|---|
| *a.* | 1. | 1. |
| *ab.* | 3. | 2. |
| *abc.* | 7. | 4. |
| *abcd.* | 15. | 8. |
| *abcde.* | 31. | 16. |
| *abcdef.* | 63. | 32. |
| *abcdefg.* | 127. | 64. |
| *abcdefgh.* | 255. | 128. |
| *abcdefghi.* | 511. | 256. |
| *abcdefghik.* | 1023. | 512. |
| *abcdefghikl.* | 2047. | 1024. |
| *abcdefghiklm.* | 4095. | 2048. |
| *abcdefghiklmn.* | 8191. | 4096. |
| *abcdefghiklmno.* | 16383. | 8192. |
| *abcdefghiklmnop.* | 32767. | 16384. |
| *abcdefghiklmnopq.* | 65535. | 32768. |

*Of*

## *Of Variations of Quantities.*

18. Having considered the Combinations and Election of Quantities, let us consider their Variations or Changes of Places.

One Quantity can admit of no Variation.

Two Quantities admit of 2 Variations; as *a b*, *b a*.

Three Quantities of 6 Changes; as, *a b c*, *a c b*, *b a c*, *b c a*, *c a b* and *c b a*; which is found by Multiplying their Indexes together, *viz.* 2 into 3; for two Quantities or Letters, admitting of 2 Changes, there are 3 several times 2 Letters; and consequently 6 Changes; as, *a*, *b*, *c*, do admit of 3 Combinations by 2 Sect. *viz.* *a b*, *a c*, *b c*: and each of these of 2 Variations; therefore the Variations of 3 Quantities are 6.

Four Quantities hath 24 Changes; for 3 Letters admitting of 6, there are four several times 3 Letters; and consequently 4 times 6 Changes, *i. e.* 24; as, *a*, *b*, *c*, *d*: here is *a b c*, *a b d*, *a c d*, and *b c d*. Now each of these, as by the last, do admit of 6 Changes: To the Changes of *a b c*, prefix *d*, then you have 6 Changes; *d a b c*, *d a c b*, *d b a c*; *d b c a*, *d c a b*, *d c b a*. So to the 6 Changes of *a b d*, prefix *c*, you will have 6 other Changes. So to the 6 Changes *a c d*, prefix *b*, you will have other 6 Changes. And again, to the 6 Changes of *b c d*, prefix *a*, you will have 6 other Changes. So in all you have 24 Changes of *a*, *b*, *c*, *d*.

Five Letters, on the same account, admits of 120, made by Multiplication of 24 in 5.

Six Letters of 720, made by Multiplication of 120 in 6.

Seven Letters of 5040, made by Multiplying 720 in 7.

19. It is required to know how many several Combinations with its Variations, there are of 3 Letters in the English Alphabet. Multiply its Combinations 2024, in its Changes 6, and

and the Product 12144 is the Number of placing 3 Letters in the Alphabet, with all their Variations.

20. But if there are 2, or more Quantities or Letters of one ſort, then divide the whole Number of Changes by the Changes of the Number of thoſe Letters, and the Quotient is the Number of Changes; as, *a*, *a*, *b*. Divide 6 its Changes by 2, the Changes of 2 Letters, the Quotient is 3, and ſo many Changes are there.

So *a a a b c* hath 20 Changes; for Divide its Changes 120, by 6, the Changes of 3, its Quotient is 29.

If there are 2, or more Letters that hav: 2 or more Letters of one ſort, then Multiply their Changes togéther for a Diviſor; as, *a a a b b*, Divide its Changes 120 by 12, the Product of 6 in 2, being the Changes of 3 and 2, the Quote is 10, its Changes.

So there are 6 Changes of *a a b b*, 210 Changes of *a a a b b c c*.

| *Quantity.* | *Variations.* |
|---|---|
| 1. | 1. |
| 2. | 2. |
| 3. | 6. |
| 4. | 24. |
| 5. | 120. |
| 6. | 720. |
| 7. | 5040. |
| 8. | 40320. |
| 9. | 36288. |
| 10. | 3628800. |
| 11. | 39916800. |
| 12. | 479001600. |
| 13. | 62270208oo. |
| 14. | 87178291200. |
| 15. | 1307674368000. |
| 16. | 20922789888000. |
| 17. | 355687537996000. |

18. 6401375683928000.
19. 121645137994632000.
20. 2432902759892640000.
21. 51090957957745440000.
22. 1124001075070399689000.
23. 25852014726619192610000.
24. 620448593438860623360000.

By this Method may be found how many several Changes there may be on each Chance on any Number of Dice; as,

*On 3 Dice.*

| *Chances.* | *Changes.* |
|---|---|
| 1, 1, 1. | 1. |
| 1, 1, 2. | 3. |
| 1, 2, 3. | 6. |

*On 4 Dice.*

| *Chances.* | *Changes.* |
|---|---|
| 1, 1, 1, 1. | 1. |
| 1, 1, 1, 2. | 4. |
| 1, 1, 2, 2. | 6. |
| 1, 1, 2, 3. | 12. |
| 1, 2, 3, 4. | 24. |

*On 6 Dice.*

| *Chances.* | *Changes.* |
|---|---|
| 1, 1, 1, 1, 1, 1. | 1. |
| 1, 1, 1, 1, 1, 2. | 6. |
| 1, 1, 1, 1, 2, 2. | 15. |
| 1, 1, 1, 1, 2, 3. | 30. |
| 1, 1, 1, 2, 2, 2. | 20. |
| 1, 1, 1, 2, 2, 3. | 60. |
| 1, 1, 1, 2, 3, 4. | 120. |
| 1, 1, 2, 2, 3, 3. | 90. |
| 1, 1, 2, 2, 3, 4. | 180. |
| 1, 1, 2, 3, 4, 5. | 360. |
| 1, 2, 3, 4, 5, 6. | 720. |

21. How many Elections are there of 10 Figures with all its Variations?

Multiply all its several Combinations with its proper Changes, then the Sum of the Products is equal to all the Elections with their Changes.

Multiply

10 in 1——————10.
45 in 2——————90.
120 in 6——————720.
210 in 24——————5040.
Multiply 252 in 120——=——30240.
210 in 720——=——151200.
120 in 5040——=——604800.
45 in 40320——=——1814400.
10 in 362880——=——2628800.
1 in 3628800——=——2628800.

9864100 = all its Elections with their ſeveral Changes.

## *Of Compoſition of Quantities.*

Although this is the moſt compoſed way, yet this is the eaſieſt to be performed; for if the compoſition of two Quantities in 10, or any other Number given (as the Alphabet) is ſought; it is but ſquaring 10 or 24, and therefore there are 100 Conjunctions of 2 Figures in 10; 576 of 2 Letters in the Alphabet.

23. If the compoſition of 3 Quantities be ſought, it is the cubing of the given Number, as, 1000, of 3 Figures. 13824 of the Letters in the Alphabet.

If the compoſition of 4 Quantities, then it is the Biquadratick power of the number; as 10000 of the Figures, 331776 of the Letters.

If of 5 Quantities, the Quadricubick power; as, 100000 of the Figures. 7962624 of the Letters.

If of 6 Quantities, the Cubocubick power; as, 1000000 of the Figures, 191102976 of the Letters; and ſo of more Quantities.

It may be Objected, that herein I am miſtaken; for in the Table

Table of Numbers there are but 900 Conjunctions of 3 Figures; for the Figures under 10, are but of one place; between 10 and 100, of 2 places,

I Answer; If in writing those numbers, you will supply what is to be understood, by filling up the void places with Cyphers, as 000, 001, 002, 003, 004, &c. 011, 012, 013, 021, 029, 075, &c. you will find there are 1000 Numbers that consist of 3 places.

How many several dispositions may there be of the 24 Letters, taking them by one and one, two and two, three and three, and so to 24?

If it had been demanded to know how many Dispositions or Compositions there are of 24 Letters, accounting each time 24, the Answer would have been,

133373577685028412444908147284377 6, The Number of several Compositions that 24 Letters may make, being the 24th. power of 24: But being we are to find all the Numbers proceeding in Geometrical Progression under it,

Let $r$ = *Ratio* = 24, and $a$ = *first Term* = 24.

$t$ = Number of Terms,

$z$ = Sum of all the Terms.

$r - 1 \,.\, a :: r^{(t)} - 1 \,.\, z$. (Dr. *Wallis* Arithm.)

that is to say, as the *Ratio* lessen'd by an Unit, is to the *first Term*, so is the *Ratio* multiplied in it self as often as is the number of *Terms* lessened by an Unit, to the Sum of all the *Terms*;

The 24th. power of 24 is

133373577685028412444908147284377 6. Which lessened by an Unite, and encreased by $\frac{1}{23}$ of it, is the Num. desired, thus;

$r^{(t)} - 1$ . 133373577685028412444908147284377 5.

$+\frac{1}{23}$ 5798851201696887497604702055 8425.

13917242888872529994251284934902200. the Number sought. Which in a *French* Author, Entituled, *Elemens des Mathematiques*; Published 1675. is miss-calculated, where he gives you the Answer, but not the manner of Operation.

24. How many ſeveral Chances are there on 2, 3, 4, 5 and 6 Dice?

According to the foregoing Rules, I ſay there are 36 chances on 2 Dice, 216 chances on 3 Dice, 1296 chances on 4 Dice, 7776 chances on 5 Dice, 46656 chances on 6 Dice.

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*A more particular Account according to the foregoing Rules of Variation, on 2, 3, and 4 Dice, followeth.*

The ſeveral Chances on 2 Dice.

| *Caſts.* | *Points.* | *Changes.* | *Sum.* |
|---|---|---|---|
| 2, 12. | 1, 1. | 1, | 1. |
| 3, 11. | 1, 2. | 2. | 2. |
| 4, 10. | 1, 3. | 2 } | 3. |
| | 2, 2. | 1 } | |
| 5, 9. | 1, 4. | 2. } | 4. |
| | 2, 3. | 2. } | |
| 6, 8. | 1, 5. | 2. } | 5. |
| | 2, 4. | 2. } | |
| | 3, 3. | 1. } | |
| 7. | 1, 6. | 2. } | 6. |
| | 2, 5. | 2. } | |
| | 3, 4. | 2. } | |

The Sum of the Chances of 2, 3, 4, 5, 6 Caſts, are 15: So likewiſe of 12, 11, 10, 9, 8 Caſts, are 15, which I have forborn to ſet down in particular, for that they are the Sum; as, 2, 3, 4, 5, 6. *Mutatis mutandis.*

To which add the Chances of 7, which are 6; their Sum is 36, the Chances of 2 Dice.

---

The Chances on 3 Dice.

| *Caſts.* | *Points.* | *Changes* | *Sum.* |
|---|---|---|---|
| 3 | 18 | 1 1 1 | 1. |
| 4 | 17 | 1 1 2 | 3. |
| 5 | 16 | 1 2 2 } | 6. |
| | | 1 1 3 } | |
| 6 | 15 | 1 1 4 } | 10. |
| | | 1 2 3 } | |
| | | 2 2 2 } | |

7 14

| | | | | | |
|---|---|---|---|---|---|
| | | 1 1 5 | 3 | | |
| | | 1 2 4 | 6 | | |
| 7 | 14 | 1 3 3 | 3 | 15. | |
| | | 2 3 3 | 3 | | |
| | | 2 2 3 | 3 | | |
| | | 1 1 6 | 3 | | |
| | | 1 2 5 | 6 | | |
| 8 | 13 | 1 3 4 | 6 | 21. | |
| | | 2 2 4 | 3 | | |
| | | 3 3 2 | 3 | | |
| | | 1 2 6 | 6 | | |
| | | 1 3 5 | 6 | | |
| 9 | 12 | 1 4 4 | 3 | 25. | |
| | | 2 3 4 | 6 | | |
| | | 2 2 5 | 3 | | |
| | | 3 3 3 | 1 | | |
| | | 1 3 6 | 6 | | |
| | | 1 4 5 | 6 | | |
| 10 | 11 | 2 3 5 | 6 | 27. | |
| | | 2 2 6 | 3 | | |
| | | 2 4 4 | 3 | | |
| | | 3 3 4 | 3 | | |

| *The Casts.* | | *Sum of the Changes* |
|---|---|---|
| 3 | 18 | 1. |
| 4 | 17 | 3. |
| 5 | 16 | 6. |
| 6 | 15 | 10. |
| 7 | 14 | 15. |
| 8 | 13 | 21. |
| 9 | 12 | 25. |
| 10 | 11 | 27 |
| | | 108. |

Which Multiplied in 2 (for the Chances, of 18, 17, 16, 15, 14, 12, & 11, are so many) produces 216, the chances of 3 Dice.

## The several Chances on 4 Dice.

| *Casts.* | | *Points.* | *Chan.* | *Sum.* |
|---|---|---|---|---|
| 4 | 24 | 1 1 1 1 | 1 | 1. |
| 5 | 23 | 1 1 1 2 | 4 | 4. |
| 6 | 22 | 1 1 1 3 | 4 | 10. |
| | | 1 1 2 2 | 6 | |
| | | 1 1 1 4 | 4 | |
| 7 | 21 | 1 1 2 3 | 12 | 20. |
| | | 1 2 2 2 | 4 | |
| | | 1 1 1 5 | 4 | |
| | | 1 1 2 4 | 12 | |
| 8 | 20 | 1 1 3 3 | 6 | 35. |
| | | 1 2 2 3 | 12 | |
| | | 2 2 2 2 | 1 | |
| | | 1 1 1 6 | 4 | |
| | | 1 1 2 5 | 12 | |
| | | 1 1 3 4 | 12 | |
| 9 | 29 | 1 2 2 4 | 12 | 56. |
| | | 1 2 3 3 | 12 | |
| | | 2 2 2 3 | 3 | |

| Casts. | Points. | Chan. | Sum. |
|---|---|---|---|
| 10 18 | 1 1 2 6 12 | | 80. |
| | 1 1 3 5 12 | | |
| | 1 1 4 4 6 | | |
| | 1 2 3 4 24 | | |
| | 1 2 2 5 12 | | |
| | 1 3 3 3 4 | | |
| | 2 2 3 3 6 | | |
| | 2 2 2 4 4 | | |
| 11 17 | 1 1 3 6 12 | | 104. |
| | 1 1 4 5 12 | | |
| | 1 2 2 6 12 | | |
| | 1 2 3 5 24 | | |
| | 1 2 4 4 12 | | |
| | 1 3 3 4 12 | | |
| | 2 2 2 5 4 | | |
| | 2 2 3 4 12 | | |
| | 2 3 3 3 6 | | |
| 12 16 | 1 1 4 6 12 | | 125. |
| | 1 1 5 5 6 | | |
| | 1 2 3 6 24 | | |
| | 1 2 4 5 24 | | |
| | 1 3 3 5 12 | | |
| | 1 3 4 4 12 | | |
| | 2 2 2 6 4 | | |
| | 2 2 4 4 6 | | |
| | 2 3 3 4 12 | | |
| | 3 3 3 3 1 | | |

| Casts | Points. | Chan. | Sum. |
|---|---|---|---|
| 13 15 | 1 1 5 6 12 | | 140 |
| | 1 2 4 6 24 | | |
| | 1 2 5 5 12 | | |
| | 1 3 3 6 12 | | |
| | 1 3 4 5 24 | | |
| | 1 4 4 4 4 | | |
| | 2 2 3 6 12 | | |
| | 2 2 4 5 12 | | |
| | 2 3 3 5 12 | | |
| | 2 3 4 4 12 | | |
| | 3 3 3 4 4 | | |
| 14 | 1 1 6 6 6 | | 146. |
| | 1 1 5 6 24 | | |
| | 1 3 4 6 24 | | |
| | 1 3 5 5 12 | | |
| | 1 4 4 5 12 | | |
| | 2 2 4 6 12 | | |
| | 2 2 5 5 6 | | |
| | 2 3 3 6 12 | | |
| | 2 3 4 5 24 | | |
| | 2 4 4 4 4 | | |
| | 3 3 3 5 4 | | |
| | 3 3 4 4 6 | | |

Casts.

| Casts. | Sum of the Chances. |
|---|---|
| 4, 24 | 1 |
| 5, 23 | 4 |
| 6, 22 | 10 |
| 7, 21 | 20 |
| 8, 20 | 35 |
| 9, 19 | 56 |
| 10, 18 | 80 |
| 11, 17 | 104 |
| 12, 16 | 125 |
| 13, 15 | 140 |
| | 575. |

Which being doubled, is 1150; to which add 146, the Chances on 14, the Sum is 1296, the Chances on 4 Dice.

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If you will take an account of the Casts whose Changes are 8, 4, & 6, you will find there are 216 Chances of double pairs, or *In and In*, as it is usually call'd in the Play on 4 Dice that usually bears that Name.

If you reckon the Chances whose Changes are 3, 4, in which are no Pairs, you will find them to be 360; and so many Chances there are where there is no Pair, and consequently there are 720 Chances where there are single Pairs.

The said number of 360 Chances having no Pair, may likewise be found by 5, 5, thus 6—3=3. with which enter the Table, and even with it in the Fourth Row, stands 15, its Combinations; which multiplied in its Changes 24, produces 360, as before.

I will give you another Mathematical Observation on Dice.

*The particular Chances on 2 Dice are,*

| | | | | | |
|---|---|---|---|---|---|
| 1, 1. | 1, 2 | 1, 3 | 14 | 15 | 16 |
| 2, 1 | 22 | 23 | 24 | 25 | 26 |
| 3, 1 | 32 | 33 | 34 | 35 | 36 |
| 4, 1 | 42 | 43 | 44 | 45 | 46 |
| 51 | 52 | 53 | 54 | 55 | 56 |
| 61 | 62 | 63 | 64 | 65 | 66 |

Here is 36 Chances the Square of 6, as before, all the Chances of 6 are placed in the lowermost and furthermost Row, so that in the Square of 5, *i. e.* 25, there is no 6. So there are 25 Chances without 6, and 11 where there is a 6.

In the Square of 4, *i. e.* 16 Chances, there is neither 5 nor 6.
In the Square of 3, *i. e.* 9, there is neither 4, 5, nor 6.
In the Square of 2, *i. e.* 4, there is neither 3, 4, 5 nor 6.
This may be applied to other Chances.

I,

| 1, | 2, | 3, | 4, | 5, | 6. |
|---|---|---|---|---|---|
| *Lat.* | *q,* | *r,* | *qq,* | *qc,* | *cc.* |
| 1, | 1, | 1, | 1, | 1, | 1. |
| 2, | 4, | 8, | 16, | 32, | 64. |
| 3, | 9, | 27, | 81, | 243, | 729. |
| 4, | 16, | 64, | 256, | 1024, | 4096. |
| 5, | 25, | 125, | 625, | 3125, | 15625. |
| 6, | 36, | 216, | 1296, | 7776, | 46656. |

This is a Table of the Powers of 6, and of the Numbers under it; and by it may be accounted how many several Chances there are on 2, 3, 4, 5, or 6 Dice, and how many there are where there is no 6, or neither 5 nor 6, or neither 4, 5, nor 6; or neither 3, 4, 5, nor 6. On 2 Dice we have done it already.

On 3 Dice, I say that there are 125 Chances where there is no 6, and 91, where there is a 6; for 216—125=91; that is to say, in the Cube of 5 there is no 6. In the Cube of 4, that is, 64, there is neither 5 nor 6; and the Remainer (it being substracted from the Cube of 6, viz. 216.) is 152, which are the Chances where there is a 5, or 6.

216—27=189, which are the Chances where there is a 4, 5, or 6, and the Cube of 3, sheweth that there are 27 Chances that have neither 4, 5, nor 6.

If it be demanded on how many Chances on 4 Dice, there is no 6, I say on 625, the Biquadrate of 5; and 1296—625=671; and in so many Chances there is a 6.

In 256 Chances there is neither 5 nor 6. 1296—256=1040, and so many Chances hath either a 5 or a 6.

In 81 Chances there is neither 4, 5, nor 6; 1296—81=1215; so many Chances hath either a 4, 5, or 6.

In 16 Chances there is neither 4, 5, nor 6. 1296—16=1280, and so many Chances hath either 3, 4, 5, or 6.

After the same manner, on 5 or 6 Dice, their Chances may be

be ſhewn; and what is ſaid here of 6, is true of any othe ſingle Chance.

Foraſmuch as all Powers of Numbers do conſiſt of ſeveral Combinations of Differences, as will appear; I will preſent you with a View of the Tables of the Powers of Numbers, as in relation to their Differences; and how (having any Power with its Differences) to find any other Power and its Differences.

1. The Differences of Squares do proceed in Arithmetical Progreſſion, and its common Difference is 2.

2. If you ſubſtract the Difference of the Differences of any Power, ſo often as the Index of the Power is, you will find they do encreaſe in Arithmetical Progreſſion, and their common Difference is found by Multiplying all the Indexes under it togethter, and in its own Index.

| *Square.* | | | | *Cube.* | | | | |
|---|---|---|---|---|---|---|---|---|
| *c.* | *b.* | | | | *b.* | | | |
| 0 | 0 | *d. Firſt Difference.* | | 0 | 0 | *d. Firſt Difference.* | | |
| 1 | 1 | 1 | *f. Common Differ.* | 1 | 1 | *f. Second Difference.* | | |
| 2 | 4 | 3 | 2. | 2 | 8 | 7 | 6 | *g. C. D.* |
| 3 | 9 | 5 | 2. | 3 | 27 | 19 | 12 | 6. |
| 4 | 16 | 7 | 2. | 4 | 64 | 37 | 18 | 6. |
| 5 | 25 | 9 | 2. | 5 | 125 | 61 | 24 | 6. |
| 6 | 36 | 11 | 2. | 6 | 216 | 91 | 30 | 6. |
| 7 | 49 | 13 | 2. | 7 | 343 | 127 | 36 | 6. |
| 8 | 64 | 15 | 2. | 8 | 512 | 169 | 42 | 6. |
| 9 | 81 | 17 | 2. | 9 | 729 | 217 | 48 | 6. |
| 10 | 100 | 19 | 2. | 10 | 1000 | 271 | 54 | 6. |

Its *Common Diſference* 6, is made by Multiplying its *Index* 3 in 2.

*b.*

| | | *b.* | *Biquadrate, or Quadriquadrate.* | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | *d.* | *Its first Difference.* | | | |
| 1 | 1 | 1 | *f.* | *Second Difference.* | | |
| 2 | 16 | 15 | 14 | *g.* | *Third Difference.* | |
| 3 | 81 | 65 | 50 | 36 | *h.* | *Common Difference.* |
| 4 | 256 | 175 | 110 | 60 | 24 | |
| 5 | 625 | 369 | 194 | 84 | 24 | |
| 6 | 1296 | 671 | 302 | 108 | 24 | |
| 7 | 2401 | 1105 | 434 | 132 | 24 | |
| 8 | 4096 | 1695 | 590 | 156 | 24 | |
| 9 | 6561 | 2465 | 770 | 180 | 24 | |
| 10 | 10000 | 3439 | 974 | 204 | 24. | |

Its *Common Difference* 24, is made by Multiplying its Index 4, in 3 in 2.

| | | *b.* | *Quadricube.* | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | *d.* | *First Difference.* | | | | |
| 1 | 1 | 1 | *f.* | *Second Difference.* | | | |
| 2 | 32 | 31 | 30 | *g.* | *Third Difference.* | | |
| 3 | 343 | 211 | 180 | 150 | *h.* | *Fourth Difference.* | |
| 4 | 1024 | 781 | 570 | 390 | 240 | *k.* | *Common Dif.* |
| 5 | 3125 | 2101 | 1320 | 750 | 360 | 120 | |
| 6 | 7776 | 4651 | 2550 | 1230 | 480 | 120 | |
| 7 | 16807 | 9031 | 4380 | 1830 | 600 | 120 | |
| 8 | 32768 | 15961 | 6930 | 2550 | 720 | 120 | |
| 9 | 59049 | 26281 | 10320 | 3390 | 840 | 120 | |
| 10 | 100000 | 40951 | 14670 | 4350 | 960 | 120. | |

Its *Common Difference* 220, is made by Multiplying its Index 5, in 4, in 3, in 2.

$\mathcal{Q}$. ſtands for Square, as $\mathcal{Q}$ 5. that is the Square of 5.

$C$. for Cube. $\mathcal{Q}\mathcal{Q}$. for the Quadriquadratick, or Biquadrate.

$\mathcal{Q}C$. for Quadricube.

Any Power with its Differences being given, to find any other Power and its Differences, let $b$ ſtand for the Power given; $d$, for its firſt Difference; $f$, for its ſecond Difference; $g$, for its third Difference; $h$, for its fourth Difference; $k$, for its fifth Difference; $m$, for its ſixth Difference.

And if the Power be greater, no more Symbols.

With the Differences of the ſides of the Powers given and ſought, enter the Figurate Table, and to the Power given $b$, add ſo many $d$'s as are in the firſt Row even with the Difference of the ſides; and ſo many $f$'s as are in the ſecond Row, and ſo many $g$'s as are in the third Row, ſo many $h$'s as are in the fourth Row, ſo many $k$'s as are in the fifth, and ſo many $m$'s as are in the ſixth Row, where occaſion requires, and the Sum is the Power ſought.

If the Power ſought, be a Squre, take only the two firſt Differences, $d$, and $f$.

If a Cube; uſe only the three firſt Differences, $d$, $f$, $g$.

If a Biquadrate; the four firſt Differences, $d$, $f$, $g$, and $h$.

If a Quadricube; the Differences $d$, $f$, $g$, $h$, and $k$.

If a Cubicube; the Differences $d$, $f$, $g$, $h$, $k$, and $m$.

The Square of 5 is $25=b$, $d=9$, $f=2$. What is the Square of 9? $9-5=4$. With it enter the Figurate Table, then $b+4d+10f=\mathcal{Q}.9$.

$25+36+20=81$.

The Cube of 5 is 27. What is the Cube of 10? Here $b=7.d=19.f=12, g=6.10-3=7$. With it enter the Table.

$b+7d+28f+84g=1000=$ Cube of 10.

$27+133+336+504=1000$.

The

The $QQ.$ of 4, is 256. What is $QQ.$ of 10. Here $b=256$. $d=671$. $f=110$. $g=60$. $h=24$. $10-4=6$. With it enter the Figurate Table. Then,

$b+6d+21f+56g+126h=QQ.10.$

$256+1050+2310+3360+3024=10000.$

The $QC.$ of 4 is 1024. What is the $QC.$ of 10. Here $b=1024$. $d=781$. $f=570$. $g=390$. $h=240$. $k=120$. $10-4=6$.

$b+6d+21f+56g+126h+25\cdot k=QC.10.$

$1024+4586+11970+21840+30240+30240=100000.$

As the Powers are thus found, so are their Differences; if it be $d$ its first Difference that is sought, then let the first Row be joyned to $f$, the second to $g$, the third to $h$, the fourth to $k$.

If it be $f$, the second Difference, then let the first Row be joyned to $g$, the second to $h$, the third to $k$.

If it be $g$, the third Difference, then let the first Row be joyned to $h$, the second to $k$, &c.

As in the Example of the Cube; $b=27$. $d=19$. $f=12$. $g=6$. What the Cube of 10 is we have found, what are the Differences?

$d+7f+289=$ the first difference of $10.=d.$

$19+84+168=271=d.$

$f+7g=$ second difference $f.$

$12+42=54.$

The Cube of 10 is 1000, what is the Cube of 20?

$b=1000$. $d=271$. $f=54$. $g=6$. $20-10=10$. With 10 enter the Figurate Table.

$b+10d+55f+220g=$ Cube of 20.

$1000+2710+2970+1320=8000.$

$d+10f+55g=d$ first difference.

$271+540+330=1141.$

$f+10g=f$ second difference.
$54+60=114$.
$b+d+f+g=$ Cube of 21.
$8000+1141+114+6=9261$.

In the Example of the Quadricube given, what are the Differences?

$d+6f+21g+56b+126k=d.$
$781+3420+8190+13440+15120=40951.$

$f+6g+28b+56k=f$
$570+2340+5040+6720=14670.$

$g+6b+21k=g$
$390+1440+1520=4350.$

$b+6k.=:$
$240+720=960$

$b+d+f+g+b+k=QC.$ 11
$100000+40951+14670+4350+660+120=161051.$

My manner of discovery hereof, was thus;

I first made these Observations on the Tables of the Differences of Powers.

1. Every Power is equal to its own Difference, and to all the Differences above it; as,

$Q.16=7+5+3+1.$ $Cub.64=37+19+7+1.$
$QQ.256=175+65+15+1.$ $QC.243=211+31+1.$

2. Every Power increased with all the Differences that stand in the same Row, is equal unto the next Power following; as,

$Q.16+7+2=Q.25.$
$Cub.\ 64+37+18+6=C.125.$
$QQ.256+175+110+60+14.$ $QQ.625.$

3. Every

3. Every Power increaſed with the Difference under its own Difference, until you come even with any Power deſired, is equal unto that Power; as, $\mathcal{Q}.\ 9+7+9+11=\mathcal{Q}.\ 36$. $C.\ 8+19+37=C.\ 64$. $\mathcal{QQ}.\ 16+65+175=\mathcal{QQ}.\ 256$.

4. Every Difference is equal unto the Difference above it, and all ſtanding even with that to the right hand, that is to be underſtood where the Table hath its full Differences.

As in the Table of Cubes, $91=61+24+6$. $127=91+30+6$.

In the Table of Quadriquadrates, $671=369+294+84+24$.

In the next place, I began to examine the Tables thus; the Square of 1 is 1, what is the Square of 5? Here $d=3.f=2$.

3 *Obſerv.* $b+5+7+9=25=\mathcal{Q}.\ 5$.

that is,

4 *Obſerv.*

| | | | |
|---|---|---|---|
| $b+5$ *i.e.* $3+2$. | $d+f.$ | $d+f.$ | 5. |
| $+7$ *i.e.* $5+2$ *i.e.* 5. | $d+f.$ | $d+2f.$ | 7. |
| | $2\ f.$ | | |
| $+9$ *i.e.* $7+2$ *i.e.* 7. | $d+2f.$ | $d+3f.$ | 9. |
| | $2.\ f.$ | $3d+6f.$ | |

So that $b+3d+6f=\mathcal{Q}.\ 5$.
$4+9+12=25$.

In the next place, I tried the Cube before mentioned thus;

$b=17\ .d=19.f=12.\ g=6.$ $\left\{\begin{array}{l} b+37+61+91+127+ \\ +169+217+271=1000. \end{array}\right.$

3 *Obſerv.* $b+37$. *i.e.* $19+12+6$. $d+f+g$. $d+f+g=37$.

4 *Obſ.*

$$\left.\begin{array}{r} 37.d+f+g. \\ 4\ Observ.\ +61.\ i.e.\ 37+18+6\ i.e.\ 18.\quad f+g. \\ 6.\qquad g. \end{array}\right\} \begin{array}{l} d+2f \\ +3g \\ =61. \end{array}$$

$$\left.\begin{array}{r} 61.d+2f+3g. \\ +91\ i.e.\ 61+24+6.\ i.e.\ 24.\quad f+2g. \\ 6.\qquad g. \end{array}\right\} \begin{array}{l} d+3f \\ +6g \\ =91. \end{array}$$

$$\left.\begin{array}{r} 91.d+3f+6g. \\ +127.\ i.e.\ 91+30+6\ i.e.\ 30.\quad f+3g. \\ 6.\qquad g. \end{array}\right\} \begin{array}{l} d+4f \\ +10g \\ =127. \end{array}$$

$$\left.\begin{array}{r} 127.d+4f+10g. \\ +169.\ i.e.\ 127+36+6\quad 36.\quad f+4g. \\ 6.\qquad g. \end{array}\right\} \begin{array}{l} d+5f+ \\ 15g= \\ 169. \end{array}$$

$$\left.\begin{array}{r} 169.d+5f+15g. \\ +217.\ i.e.\ 169+42+6\ i.e.\ 42.\quad f+5g. \\ 6.\qquad g. \end{array}\right\} \begin{array}{l} d+6f \\ +21g \\ =217 \end{array}$$

$$\left.\begin{array}{r} 217.d+6f+21g. \\ +271.\ i.e.\ 217+48+6.\ i.e.\ 48.\quad f+6g. \\ 6.\qquad g. \end{array}\right\} \begin{array}{l} d+7f \\ +28g \\ =271. \end{array}$$

$$7d+28f+84g.$$

$b+7d+28f+84g=$ Cube of 10.
$27+133+336+504=1000.$

In the Biquadratick Power, I found after the same manner, that the $QQ.$ of 3, that is, $81=b.$

$$81.$$
$$+7d+28f+84g+210h=QQ.\ 10.$$
$$455+1400+3024+5040=10000.$$

Here $d=65.\ f=50.\ g=36.\ h=24.$

From

From whence I obſerved that the Figures joyned with the Differences, were all Figurate Numbers, and proceeding in order.

Thoſe joyned with the firſt Difference, *d*, to be Lateral Numbers.

Thoſe joyned with the ſecond Difference, *f*, to be Triangular Numbers.

Thoſe with the third Difference, *g*, to be Pyramidal Numbers.

Thoſe with the fourth Difference, *h*, to be Triangulo-Pyramidal Numbers.

Thoſe with the fifth Difference, *k*, to be Pyrami-pyramidal Numbers.

But as in Combinations, in caſe where there is no Figurate Table, or the Number of the Difference of the ſides do exceed the Table, the Powers may be thus found;

Take the Difference of the ſides of the Powers given and ſought, and increaſe it by an Unite as often as the Index of the Power is, ſave one, placing them with the Sign of Multiplication × between them for a Dividend; then place an Unite with the like Number of Figures increaſing by an Unite with the Sign of Multiplication × between them for a Diviſor; then prepare the Terms, by dividing the Dividend and Diviſor by each ſeveral Number in the Diviſor, and the Diviſor will be brought to an Unite; then multiply what is remaining in the *Dividend*, for a *Quotient*.

The *QQ.* of 10 is 10000; what is the *QQ.* of 20?

Where, $h=10000$. $d=3439$. $f=974$. $g=194$. $k=24$.

$20-10=10$.

$4\times3\times2\times1.$ )$10\times11\times12\times13$(
$5$ $4$

| *The Factors of the Quot.* | | *The Quot.* | | $b.$ 10000. |
|---|---|---|---|---|
| 10 | | 10 $d.$ | $10\times3439.$ | 34390. |
| $5\times11$ | $=$ | 55 $f.$ | $55\times974.$ | 53570. |
| $5\times11\times4$ | | 220 $g.$ | $220\times204.$ | 44880. |
| $5\times11\times13.$ | | 715 $b.$ | $715\times24.$ | 17160. |
| The $Q.Q.$ of 20. | | | | 160000. |

This differs from the manner of operation in Combinations in this, that there the Quantities do decreaſe, and here they do increaſe in Arithmetical Progreſſion.

The $Q.C.$ of 5 is $3125 = b.$ $2101 = d$, the firſt Difference 31; $1320 = f.$ $750 = g.$ $360 = b.$ $120 = k.$ what is the $Q.C.$ of 105. $105-5=100.$

$5\times4\times3\times2\times1$ )$100\times101\times102\times103\times104$(

| *The Factors in the Quot.* | | | |
|---|---|---|---|
| 100 | | | 100. |
| $50\times101$ | $=$ | | 5050. |
| $50\times101\times34$ | | | 171700. |
| $25\times101\times17\times103$ | $=$ | | 4421275. |
| $5\times101\times17\times103\times104$ | | | 91962520. |

| that is, | $b.$ | | 3125. |
|---|---|---|---|
| 100 $d.$ i.e. | $100\times2101$ | $=$ | 210100. |
| 5050 $f.$ i.e | $5050\times1320$ | | 6666000. |
| 171700 $g.$ i.e. | $171700\times750$ | | 128775000. |
| 4421275 $b.$ i.e. | $4421275\times360.$ | | 1591659000. |
| 91962520 $k.$ i.e. | $91962520\times120.$ | | 11035502400. |
| | | | 12762815625 $= Q.C.$ 105. |

This

This way shews how any Figurate Number for any Latus may be found, and any Figurate Table may be examined; as what are the Figurate Numbers for 30.

11×10×9×8×7×6×5×4×3×2×1 ) 30×31×32×33×34×35×36×37×38×39×40.

| *The Factors of the Quotients are* | *The Quotients.* |
|---|---|
| 1 \| 1 | 1 |
| 2 \| 30 | 30 |
| 3 \| 15×31 | 465 |
| 4 \| 5×31×32 | 4960 |
| 5 \| 5×31×8×33 | 40920 |
| 6 \| 31×8×33×34 | 278256 |
| 7 \| 31×8×11×17×35 | 1623160 |
| 8 \| 31×8×11×17×5×36 | 8347680 |
| 9 \| 31×11×17×5×36×37 | 38608020 |
| 10 \| 31×11×17×5×4×37×38 | 163011640 |
| 11 \| 31×11×17×2×37×38×39 | 635745396 |
| 12 \| 31×17×2×37×38×39×40 | 2311801440 |

Agreeing with my Figurate Table.

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## *A Demonstration of the Rules in the* 9th *and last Paragraphs.*

ALthough the Rule here given for the finding a Figurate Nuuber, and the Rule in the *9th.* Paragraph do differ in this, that the Quantities there do decrease, here increase in Arithmetical Progression, yet they do both spring from the last consequence cited by me out of Monsieur *Pascall* on Figurate Numbers.

For by that conſequence any Figurate Number with its place being given, it is eaſie to find its next Number in a Diagonal Line either upward or downward.

As 165 being the 9*th* Number in the 4*th* Row, what is the 8*th* Number in the 5*th* Row? Say, as 4 is to 8, ſo is 165 to 330, the Number required.

What is the 10*th* Number in the 3*d* Row? Say, as 9 is to 45, ſo is 165 to 55, the Number deſired.

3060 being in the 15*th* Line in the 5*th* Row, what are the Numbers joyinng to it in the ſame Diagonal Line? Say, as 5. 14:: 3060. 8568. being in the 14*th* Line in the 6*th* Row. Again, 15. 4:: 3060. 816. being the 16*th* Number in the 4*th* Row.

And conſequently, the place of any Figurate Number being given, that Figurate Number may be thus found;

First, Find the Natural or Lateral Number in the ſame Diagonal Line, then the next Number to the Lateral in the ſame Diagonal Line, then the next to that, until you come to the Number deſired.

The Lateral Number is known by ſubſtracting 2 from the Sum of both places; that is from the Number of the Rows and of the Lines.

As what is the 5*th* Number in the 3*d* Row?

The Sum of 5 and 3 is 8; from which ſubſtract 2, there remains 6, the Lateral Number of the ſame Diagonal Line. Then ſay, as 2 is to 5, ſo is 6 to 15, the Number required.

What is the 4*th* Number in the 4*th* Row? Say, $4+4=8$, leſſened by 2, is equal to 6, its Lateral Number, then ſay as before, 2. 5:: 6. 15. Again ſay, 3. 4:: 15. 20, the Number ſought for: Or thus;

2. 6:: 5. $\frac{6+5}{2}=15$. Again, 3. 4:: $\frac{6+5}{2}$ (15) $\frac{6\times5\times4}{3+2}$ $=20$.

What

What is the *6th* Number in the *6th* Row?

$6+6=12$, leſſened by $2=10$, its Lateral Number, then find the next to its Lateral Number in the ſame Diagonal, and ſo continue working until you ſhall come to the Number required, thus; $2.9.10::\frac{10\times9}{2}=45$, the *9th* Number in the *3d* Row.

Again, $3.8::\frac{10\times9}{2}(45)\frac{10\times9\times8}{3\times2}=120$. the *8th* number in the *4th* Row.

Again, $4.7::\frac{10\times9\times8}{3\times2}(120)\frac{10\times9\times8\times7}{4\times3\times2}=210$ the *7th* Number in the *5th* Row.

Again, $5.6::\frac{10\times9\times8\times7}{4\times3\times2}(210)\frac{10\times9\times8\times7\times6}{5\times4\times3\times2}$ $=252$, the Number ſought. The very Examples of the Rule delivered in the *9th* Paragraph to find 2, 3, 4, 5, and 6 Quantities in 10.

What are the *6th* Numbers in the *3d*, *4th*, and *5th* Rows?

Theſe ſtanding in ſeveral Diagonal Lines, muſt be wrought ſeverally.

$6+3=9.\ -2=7.$ its Lateral Number: $2.6::7.\frac{6\times7}{2}=21$, the Number deſired in the *3d* Row.

$6+4=10-2=8$, its Lateral Number: $2.8::7.\frac{7\times8}{2}=28$. the *7th* Number in the *3d* Row.

Again, $3.6::\frac{7\times8}{2}(28).\frac{6\times7\times8}{3\times2}=56$, the Number required in the *4th* Row.

$6+5=11-2=9$, its Lateral Number, $2.9::8.\frac{8\times9}{2}=36$, the *8th* Number in the *3d* Row.

Again, $3 . 7 :: \frac{8 * 9}{2} (36) \frac{7 * 8 * 9}{3 * 2} = 84$. the 7*th* Number in the 4*th* Row.

Again, $4 . 6 :: \frac{7 * 8 * 9}{3 * 2} (84) \frac{6 * 7 * 8 * 9}{4 * 3 * 2} = 126$, the Number desired in the 5*th* Row.

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 6 | $\frac{6 * 7}{2} = 21.$ | $\frac{6 * 7 * 8}{3 * 2} = 56.$ | $\frac{6 * 7 * 8 * 9}{4 * 3 * 2} = 126.$ |

6

$\frac{6 * 7 * 8 * 9 * 10}{5 * 4 * 3 * 2} = 252.$

The Method of my General Rule to find any Figurate Number; then by dividing the *Diviſor* and *Dividend* by each Number in the *Diviſor*, the *Diviſor* is brought to an *Unite*; 15 *Euc.* 5.

# Here follows a Table of Figurate Numbers.

| Units. | Laterals. | Triangular. | Pyramidal. | Triangulipyramidal. | Pyrami-pyramidal. |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 3 | 6 | 10 | 15 | 21 |
| 1 | 4 | 10 | 20 | 35 | 56 |
| 1 | 5 | 15 | 35 | 70 | 126 |
| 1 | 6 | 21 | 56 | 126 | 252 |
| 1 | 7 | 28 | 84 | 210 | 462 |
| 1 | 8 | 36 | 120 | 330 | 792 |
| 1 | 9 | 45 | 165 | 495 | 1287 |
| 1 | 10 | 55 | 220 | 715 | 2002 |
| 1 | 11 | 66 | 286 | 1001 | 3003 |
| 1 | 12 | 78 | 364 | 1365 | 4368 |
| 1 | 13 | 91 | 455 | 1820 | 6188 |
| 1 | 14 | 105 | 560 | 2380 | 8568 |
| 1 | 15 | 120 | 680 | 3060 | 11628 |
| 1 | 16 | 136 | 816 | 3876 | 15504 |
| 1 | 17 | 153 | 969 | 4845 | 20349 |
| 1 | 18 | 171 | 1140 | 5985 | 26334 |
| 1 | 19 | 190 | 1330 | 7315 | 33649 |
| 1 | 20 | 210 | 1540 | 8855 | 42504 |
| 1 | 21 | 231 | 1771 | 10626 | 53130 |
| 1 | 22 | 253 | 2024 | 12650 | 65780 |
| 1 | 23 | 276 | 2300 | 14950 | 80730 |
| 1 | 24 | 300 | 2600 | 17550 | 98280 |
| 1 | 25 | 325 | 2925 | 20475 | 118755 |
| 1 | 26 | 351 | 3276 | 23751 | 142506 |
| 1 | 27 | 378 | 3654 | 27405 | 169911 |
| 1 | 28 | 406 | 4060 | 31465 | 201376 |
| 1 | 29 | 435 | 4495 | 35960 | 237336 |
| 1 | 30 | 465 | 4960 | 40920 | 278256 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 7 | 8 | 9 |
| 28 | 36 | 45 |
| 84 | 120 | 165 |
| 210 | 330 | 495 |
| 462 | 792 | 1287 |
| 924 | 1716 | 3003 |
| 1716 | 3432 | 6435 |
| 3003 | 6435 | 12870 |
| 5005 | 11440 | 24310 |
| 8008 | 19448 | 43758 |
| 12376 | 31824 | 75582 |
| 18564 | 50388 | 125970 |
| 27132 | 77520 | 203490 |
| 38760 | 116280 | 319770 |
| 54264 | 170544 | 490314 |
| 74613 | 245157 | 735471 |
| 100947 | 346104 | 1081575 |
| 134596 | 480700 | 1562275 |
| 177100 | 657800 | 2220075 |
| 230130 | 888030 | 3108105 |
| 296010 | 1184040 | 4292115 |
| 376740 | 1560780 | 5852925 |
| 475020 | 2035800 | 7888725 |
| 593775 | 2629575 | 10668310 |
| 736281 | 3365856 | 13884156 |
| 906192 | 4272048 | 18156204 |
| 1107568 | 5379616 | 23535820 |
| 1344904 | 6724520 | 30260340 |
| 1623160 | 8347680 | 38608920 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 10 | 11 | 12 |
| 55 | 66 | 78 |
| 220 | 286 | 364 |
| 715 | 1001 | 1365 |
| 2002 | 3003 | 4368 |
| 5005 | 8008 | 12376 |
| 11440 | 19448 | 31824 |
| 24310 | 43758 | 75582 |
| 48620 | 92378 | 167960 |
| 92378 | 184756 | 352716 |
| 167960 | 352716 | 705492 |
| 293930 | 646646 | 1352078 |
| 497420 | 1144066 | 2496144 |
| 817190 | 1961256 | 4457400 |
| 1307504 | 3268760 | 7726160 |
| 2042975 | 5311735 | 13037895 |
| 3124550 | 8436285 | 21474180 |
| 4686825 | 13123110 | 34597290 |
| 6906900 | 20030010 | 54672300 |
| 10015005 | 30045015 | 84672315 |
| 14307150 | 44352165 | 129024480 |
| 20160075 | 64512240 | 193536720 |
| 28048800 | 92561040 | 286097760 |
| 38567100 | 131128140 | 417125900 |
| 52451256 | 183579396 | 600705296 |
| 70607460 | 254186856 | 854992152 |
| 94143280 | 348330136 | 1203322198 |
| 124403620 | 472733756 | 1676016044 |
| 163011640 | 635745396 | 2311801440 |

It is a chief property of these Figurate Numbers, That any Number in any Row is equal to the Sum of those in the preceding Row that stands even with and above it.

As 15 in the 3*d* Row = 5+4+3+2+1 in the second Row.

So 35 in the 4*th* Row = 15+10+6+3+1, in the third Row.

So 70 = in the 5*th* Row = 35+20+10+4+1, in the 4*th* Row.

So 126 = 70+35+15+5+1, *&c.* And therefore any Number in the Figurate Table is equal unto that above it, and to that even with it in the preceding Row; as 3 = 1+2. 10 = 4+6. 126 = 70+56. 1001 = 715+286. 462 = 252+210.

*The Construction of the former Table.*

Every Figurate Number is made by adding the Number preceding it to that above it; as in the 3*d* Row, 3 = 2+1. 15 = 5+10 66 = 11+55. In the 4*th* Row, 35 = 15+20. 220 = 55+165. 680 = 120+560. In the 7*th* Row. 28 = 21+7. 210 = 126+84. 5005 = 2002+3003. From whence Monsieur *Pascall* in a Treatise on this Subject, called *Triangular Arithmetick*, hath drawn 19 several Consequences; whereof these following are the choicest.

1. Every Figurate Number is equal unto the Number preceding it, and all above it in the same Row; as in the 4*th* Row, 20 = 10+6+3+1. in the 6*th* Row, 126 = 70+35+15+5+1. in the 7*th* 28 = 21+6+1.

2. Every Figurate Number is equal to the Number above it, and all preceding it in the same Line; as in the 3*d* Row, 15 = 10+4+1. 10 = 6+3+1. in the 6*th* Row, 21 = 6+5+4+3+2+1. 126 = 56+35+20+10+4+1.

3. If the Numbers of the Lines, and Rows of any Figurate are unequal, by interchanging the said Numbers, you will find the said Figurate in another place; as the 2*d* Number in the 6*th* Row, and the 6*th* Number in the 2*d* Row, are

equal to 6; ſo the 5*th* Number in the 3*d* Row, and the 3*d* Number in the 5*th* Row, are equal to 15.

4. The Figurate Numbers in every Diagonal Line are double the Figurate Numbers in the preceding Diagonal Line; as, 1+4+6+4+1, is double 1+3+3+1, which is double 1+2+1, and that is double 1+1.

5. Any two Figurate Numbers ſtanding together in a Diagonal Line, are in proportion one to another, as their places are reſpectively diſtant from their proper Line of Unites incluſively, that is, accounting the place of the lower Number from the collateral Line of Unites, and the place of the upper Number from the Capital Line of Unites; as 5 in the Second Row, and 10 in the Third Row, are in proportion one to the other, as 2 and 4; for that 5 is in the Second Row, accounting from the collateral Line of Unites, and 10 is in the 4*th* Line, accounting from the upper Line of Units; ſo 35 in the 5*th* Row, and 21 in the 6*th* Row, are in proportion, as 5 to 3. So 56. 70::4.5. So 28. 56::3.6. So 330. 462::5.7. So 300.2300::3.23.

A Continuation of the former Figurate Table to 8 Rows only, beginning at 31, and ending at 100; in regard the other Four Rows would ſwell too much.

| Units. | Lateral. | Triangular. | Pyramidal. | Triangulipyramidal. | Pyramipyramidal. |
|---|---|---|---|---|---|
| 1 | 31 | 496 | 5456 | 46376 | 324632 |
| 1 | 32 | 528 | 5984 | 52360 | 376992 |
| 1 | 33 | 561 | 6545 | 58905 | 435897 |
| 1 | 34 | 595 | 7140 | 66045 | 501942 |
| 1 | 35 | 630 | 7770 | 72815 | 575757 |
| 1 | 36 | 666 | 8436 | 82251 | 658008 |
| 1 | 37 | 703 | 9139 | 91390 | 749398 |
| 1 | 38 | 741 | 9880 | 101170 | 850668 |
| 1 | 39 | 780 | 10660 | 111930 | 962598 |
| 1 | 40 | 820 | 11480 | 123410 | 1086008 |
| 1 | 41 | 861 | 12341 | 135751 | 1221756 |
| 1 | 42 | 903 | 13244 | 148995 | 1370754 |
| 1 | 43 | 946 | 14190 | 163185 | 1533939 |
| 1 | 44 | 990 | 15180 | 178365 | 1712304 |
| 1 | 45 | 1035 | 16215 | 194580 | 1906884 |
| 1 | 46 | 1081 | 17296 | 211876 | 2118760 |
| 1 | 47 | 1128 | 18424 | 230300 | 2349060 |
| 1 | 48 | 1176 | 19600 | 249900 | 2596960 |
| 1 | 49 | 1225 | 20825 | 270725 | 2869685 |
| 1 | 50 | 1275 | 22100 | 292825 | 3162510 |
| 1 | 51 | 1326 | 23426 | 316251 | 3478761 |
| 1 | 52 | 1378 | 24084 | 341055 | 3819816 |
| 1 | 53 | 1431 | 26235 | 367290 | 4187106 |
| 1 | 54 | 1485 | 27720 | 395010 | 4582116 |
| 1 | 55 | 1540 | 29260 | 424270 | 5006386 |
| 1 | 56 | 1596 | 30856 | 455126 | 5461512 |
| 1 | 57 | 1653 | 32509 | 487635 | 5949147 |
| 1 | 58 | 1711 | 34220 | 521855 | 6471002 |
| 1 | 59 | 1770 | 35990 | 557845 | 7028847 |
| 1 | 60 | 1830 | 37820 | 595665 | 7624512 |

| | |
|---|---|
| 1947792 | 10295472 |
| 2324784 | 12620256 |
| 2760681 | 15380937 |
| 3261623 | 18643560 |
| 3638380 | 22481930 |
| 4496388 | 26978328 |
| 5245786 | 32224114 |
| 6096454 | 38320568 |
| 7059052 | 45379620 |
| 8145060 | 53524680 |
| 9366819 | 62891499 |
| 10737573 | 73629070 |
| 12271512 | 85900584 |
| 13983816 | 99884400 |
| 15890700 | 115775100 |
| 18009460 | 133784560 |
| 20358520 | 154143080 |
| 22957480 | 177100560 |
| 25827165 | 202927725 |
| 28989675 | 231917400 |
| 32468436 | 264385836 |
| 36288252 | 300674088 |
| 40475358 | 341149446 |
| 45057404 | 386206920 |
| 50063860 | 436270780 |
| 55525372 | 491796152 |
| 61474519 | 553270671 |
| 67945521 | 621216192 |
| 74974368 | 696190560 |
| 82598880 | 778789440 |

| Units. | Lateral. | Triangular. | Pyramidal. | Triæguli-pyramidal. | Pyrami-pyramidal. |
|---|---|---|---|---|---|
| 1 | 61 | 1891 | 39711 | 635376 | 8259888 |
| 1 | 62 | 1953 | 41664 | 677040 | 8936928 |
| 1 | 63 | 2016 | 43680 | 720720 | 9657648 |
| 1 | 64 | 2080 | 45760 | 765480 | 10424128 |
| 1 | 65 | 2145 | 47905 | 814365 | 11238513 |
| 1 | 66 | 2211 | 50116 | 864501 | 12103014 |
| 1 | 67 | 2278 | 52394 | 916895 | 13019909 |
| 1 | 68 | 2346 | 54740 | 971635 | 13991544 |
| 1 | 69 | 2415 | 57155 | 1028790 | 15020334 |
| 1 | 70 | 2485 | 59640 | 1088430 | 16108764 |
| 1 | 71 | 2556 | 62196 | 1150626 | 17259390 |
| 1 | 72 | 2628 | 64824 | 1215450 | 18474840 |
| 1 | 73 | 2701 | 67525 | 1282975 | 19757815 |
| 1 | 74 | 2775 | 70300 | 1353275 | 21111090 |
| 1 | 75 | 2850 | 73150 | 1426425 | 22537515 |
| 1 | 76 | 2926 | 76076 | 1502501 | 24040016 |
| 1 | 77 | 3003 | 76079 | 1581580 | 25621596 |
| 1 | 78 | 3081 | 82160 | 1663740 | 27285336 |
| 1 | 79 | 3160 | 85320 | 1749060 | 29034396 |
| 1 | 80 | 3240 | 88590 | 1837620 | 30672016 |
| 1 | 81 | 3321 | 91881 | 1929501 | 32801517 |
| 1 | 82 | 3403 | 95284 | 2024785 | 34826302 |
| 1 | 83 | 3486 | 98770 | 2123555 | 36949857 |
| 1 | 84 | 3570 | 102340 | 2225895 | 39175752 |
| 1 | 85 | 3655 | 105995 | 2331890 | 41507642 |
| 1 | 86 | 3741 | 109736 | 2441626 | 43949268 |
| 1 | 87 | 3828 | 113564 | 2555190 | 46504458 |
| 1 | 88 | 3916 | 117480 | 2672670 | 49177128 |
| 1 | 89 | 4005 | 121485 | 2794185 | 51971283 |
| 1 | 90 | 4095 | 125580 | 2891985 | 54891018 |

| | |
|---|---|
| 90858768 | 869648208 |
| 99795696 | 969443904 |
| 109453344 | 1078887248 |
| 119877472 | 1198774720 |
| 131115985 | 1329890705 |
| 143218999 | 1473109704 |
| 156338908 | 1629348612 |
| 170230452 | 1749579064 |
| 185250786 | 1984829850 |
| 201359550 | 2186189400 |
| 218618940 | 2404808340 |
| 237093780 | 2641902120 |
| 256851595 | 2897753715 |
| 277962685 | 3176716400 |
| 300500200 | 3477216600 |
| 324540216 | 3801756816 |
| 350161812 | 4151918628 |
| 377471480 | 4529365776 |
| 406481544 | 4935847320 |
| 437353560 | 5373200880 |
| 470155077 | 5843355957 |
| 504981379 | 6348337336 |
| 541931236 | 6890268572 |
| 581106988 | 7471375560 |
| 622614630 | 8092990190 |
| 666563898 | 8760554088 |
| 713068356 | 9473622444 |
| 762245484 | 10235867928 |
| 814216767 | 11050084695 |
| 869107785 | 11919192480 |

| Units. | Lateral. | Triangular. | Pyramidal. | Triangulipyramidal. | Pyrami-pyramidal. |
|---|---|---|---|---|---|
| 1 | 91 | 4186 | 129766 | 3049501 | 57940519 |
| 1 | 92 | 4278 | 134044 | 3183545 | 61124064 |
| 1 | 93 | 4371 | 139415 | 3321960 | 64446024 |
| 1 | 94 | 4465 | 142880 | 3464840 | 67910864 |
| 1 | 95 | 4560 | 147440 | 3612200 | 71523144 |
| 1 | 96 | 4656 | 152096 | 3764376 | 75287520 |
| 1 | 97 | 4753 | 156849 | 3921225 | 79208745 |
| 1 | 98 | 4851 | 161700 | 4082925 | 83921670 |
| 1 | 99 | 4950 | 166650 | 4249575 | 87541245 |
| 1 | 100 | 5050 | 171700 | 4421275 | 91962520 |

## *Examination.*

7*6*5*4*3*2*1) 100*101*102*103*104*105*106.

| | *Factors of the Quote.* | | *Quotient.* |
|---|---|---|---|
| 2 | 50*101 | | 5050 |
| 3 | 50*101*34 | | 171700 |
| 4 | 25*101*17*103 | = | 4421275 |
| 5 | 5*101*17*103*104 | | 91962520 |
| 6 | 5*101*17*103*52*35 | | 1609344100 |
| 7 | 5*101*17*103*52*5*106 | | 24370067800 |

Being the Figurate Numbers of the { 3, 4, 5, 6, 7, 8 } Order of 190.

9270.

| | |
|---|---|
| 927048304 | 128462140784 |
| 988172368 | 138344131752 |
| 1052618391 | 148870315 44 |
| 1120529256 | 160075608 00 |
| 1192052400 | 171916122 00 |
| 1267339920 | 184069531 20 |
| 1346548665 | 198135017 55 |
| 1429840335 | 212433421 20 |
| 1517381580 | 227607237 00 |
| 1609344100 | 243700678 00 |

As the Table of Figurate Numbers may thus be examined, so if there be any miſtake in the Calculation of the Tables, you may find what it will amount unto at any place required; for all Figures in the ſame Row or Line have the ſame Miſtake: Thoſe in the next Row or Line do proceed as natural Numbers; thoſe in the third Row or Line incluſively; as *Triangulars:* thoſe in the *4th.* as *Pyramidals*; and ſo forward.

In the *6th* Number in the *4th* Row, ſuppoſe there is an Unite too much, 57, for 56, I would deſire to know what ſuch Miſtake will amount unto in the *12th.* Row, as alſo in the *100th* Number in the *8th* Row.

Take the Difference of the Rows, and the Difference of the Lines, and add an Unite to each, the Figurate Number belonging to that Row and Line is the Number deſired; as,

12 — 4 = 8 + 1 = 9. Row } The *25th* Number in the *9th*
30 — 6 = 24 + 1 = 25. Line } Row, *viz.* 1051 8300 is the Number required; ſo that the *30th* Number will be 2322319740, inſtead of 2311801440.

In the other Question,

100—6=94+1=95. Line } The 95*th* Number in the 5*th*
8—4=4+1=5. Row } Row, *viz.* 3612280, is the Exceſs that the 10*cth* Number in the 8*th* Row will be more than it ought: If the Miſtake be in the firſt place more than an Unite, you muſt multiply the Figurate found in that Number.

If any falſe Figurate Number be given, to find where the Error is, if it proceed only from the Miſtake of an Unit; In the 10*th* Row and 25*th* Line, the Figurate is 38567100, and in a falſe Table it is 36744200, occaſioned only by the miſtake of an Unite; but where I would deſire to know.

Take the Differences of the Figurates 1771100, which you will find to be the 20*th* Number in the 7*th* Row. Then,

Take the Difference of the Rows, and Difference of the Lines, and add an Unit to them, you have the place of the Error.

25—20=5+1=6. Line } The Error is in the 6*th* Number
10—7=3+1=4. Row } in the 4*th* Row, *viz.* 57 for 56.

If there be but one Error committed, the Difference of the true and falſe Figurates is alwayes a Figurate or a Compound of it.

I find in Two Treatiſes, one Entituled *Cardanus Promotus*, another about *Cubick Equations*, of the moſt Learned Mr. *Thomas Baker*, Miniſter of *Bishop Nympton* in *Devonſhire*, my Worthy Friend, a Perſon endued with profound Skill in *Algebra*, &c. whoſe pains on theſe Arguments, are now at the Preſs, and will be ſucceeded by more of the ſame kind, a Table of new Figurates, where the 6*th* Number in the 5*th* Row is 182, in this 126. I do deſire to know where the firſt Difference of the Tables is, from whence ſpring all the other Differences.

| | | | | | |
|---|---|---|---|---|---|
| 1, | 2, | 2, | 2, | 2, | 2. |
| 1, | 3, | 5, | 7, | 9, | 11. |
| 1, | 4, | 9, | 16, | 25, | 36. |
| 1, | 5, | 14, | 30, | 55, | 91. |
| 1, | 6, | 20, | 50, | 105, | 196. |
| 1, | 7, | 27, | 77, | 182, | 378. |

By the foregoing Rule, 182—126=56, which is the 6*th* Number in the 4*th* Row.

6—

6—6=0+1=1. Line } So that the first Difference is the
5—4=1+1=2. Row } first Number in the 2d Row, there being 2 instead of 1.

By the common Table of Figurates, according to the foregoing Rule, this may be made by adding to any Figurate the Number preceding it in the same Line. As,
What is the 4th Number in the 3d Row? I say, 10+4=14.
What is the 5th Number in the 4th Row? I say, 35+15=50.
252+126=378, the 6th Number in the 6th Row: 210+55 =275. =10th Number in the 4th Row.

# APPENDIX.

## *To the End of* Pag. 21, *may be further added,*

THat the Sum of all the Chances on 1, 3, or more Dice, and the Chances on the Regular Bodies, *viz.* the *Tetrahedron*, *Octahedron*, *Dodecahedron*, and *Icosahedron*, being marked on their sides as Dice, not exceeding the number of their sides, are Figurate Numbers, and if the Sum of their Chances do exceed their sides, then they are not Figurates; they beginning at an *Unit*, do proceed on until they do come to their middle Chance, then decreasing in the same manner, they do end in an *Unit*, whether it be on 2, 3, 4, or more Dice, or such Bodies: The Chances on 2 Dice, or 2 such Bodies, are natural Numbers; on 3 Dice, or 3 such Bodies, are triangular Numbers; on 4, are Pyramidal, and so forward.

So that in a *Tetrahedron*, the Sums of the 4 first Chances are Figurate Numbers; in a Cubick Die, the Sums of the six first Chances; in an *Octahedron*, the Sums of the 8 first; in a *Dodecahedron*, of the 12 first; and in an *Icosahedron*, the 20 first Chances are Figurates.

As in 5 *Tetrahedrons*, the Chances are 5,4,5,6,7,8, 9, 10,11,12.
Their Sums are 1,3,6,10,12,12,10,6, 3, 1.

The first and last 4 Figures of their Sums are Triangular Numbers; 12, which is the Sum of the Chances, 7, is not Figurate; for that 1+1+5=7. and the *Tetrahedron* having but 4 sides, and consequently no 5, the Changes of 1+1+5 =7 are 3; which being substracted from the next Figurate in the same Row, *viz.* 15, rest 12; as before.

So

So on the 3 Cubick Dice,

The Chances are 3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18.

Their Sum 1,3,6,10,15,21,25,27,27,25,21,15,10,6,3,1

The first and last six are Triangulars; 25 and 27 are not Figurates, for that $1+1+7=9$ (there being not any 7 on any one Die) cannot be on the Dice, and consequently its Changes 3, must be deducted from the next Figurate 28, remains 25, as before. Again, $1+1+8=10$, and $1+2+7=10$, neither of which can be on the Dice, there being neither 7 nor 8, their Changes 3 and $6=9$, must be deducted from 36, the 8*th* Number in that Row; then you have 27, as before.

So on 4 Dice; the first and last six Chances are Pyramidal Numbers, *viz.* 1, 4, 10, 20, 35, 56; the other, 80, 104, 125, 140, and 146 are not on the same reason.

If they had been all Figurates, their Sum would have been $715+715+286=1716$, exceeding 1296, the Chances on 4 Dice by 420.

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Upon a Re-examination of the *Table*, these Errors were found, which the Reader is desired to Correct thus.

*Range* 7, *Latus* 21, 230230. *R.* 9. *L.* 22. 4292145. *L.* 25. 105183co. *L.* 30. 38608020. *R.* 12. *L.* 12. 705432.

In the Continuation.

*Range* 4. *Lat.* 77. 79079. *L.* 80. 88560. *R.* 5. *L.* 38. 101270. *L.* 89. 2794155. *L.* 90. 2919735. *R.* 6. *L.* 41. 1221759. *L.* 48. 2598960. *L.* 80. 3872016. *R.* 7. *L.* 67. 1562389 8. *L.* 78. 377447148. *R.* 8. *L.* 63. 1078897248. *L.* 68. 1799579064. *L.* 95. 1719961320. *L.* 97. 1981350785.

## *Advertiſement.*

THere is newly Publiſhed an ingenious ſmall Tract, Entituled, *Artificial Verſifying*; which ſheweth to any one of ordinary capacity, though he underſtands not one word of *Latin*, or what a Verſe means, how, by the help of 6 Tables, to make hundreds of *Hexameter Verſes*, which ſhall be true *Latin*, true Verſe, and Senſe. The Conceit whereof is, that each Table contains 9 ſeveral Words, placed in Letters under a handſom diſguiſe; which are theſe:

Table.

1. *Impia, ſordida, aſpera, turbida, triſtia, horrida, turpia, peſſima, perfida.*
2. *Dicta, facta, fata, bella, jura, vota, verba, dona, damna.*
3. *Mihi, tibi, iniquam, viro, malis, vides, reor, aliis, ſcio.*
4. *Prædicunt, procurant, monſtrabunt, promittunt, portabunt, cauſabunt, concedunt, producunt, confirmant.*
5. *Sidera, ſomnia, pignora, fœdera, crimina, tempora, dogmata, jurgia, poſula.*
6. *Multa, quædam, certa, tamum, planè, ſola, prava, ſemper, ſæpe.*

So that if you take any one word out of each Line, you will have a true *Hexameter* Verſe.

It will be required to know how many ſeveral Verſes by theſe Lines may be made? I Anſwer, according to the fore going Rule, *pag.* 16. of *Compoſition of Quantities* (without taking any notice of the permutation of places, that may be; for you may change the places of all words in the firſt unto the 5*th* Line, and 5 words of the 6*th* into the ſecond Line) there may be made 531441 Verſes, it being the Cubo-cube of 9; namely,

ly, above 30 times as many Verſes as are in *Virgil*; for in his Works Printed at *Cambridge*, 1632, there are 492 Pages, and each full Page contains 30 Verſes, ſo that there are not in *Virgil* above 17712 Verſes. For if you join the Words in the firſt Line with thoſe in the ſecond, you will have 81 ſeveral Combinations of two Words, the Square of 9; and if to thoſe you add each Word in the third Line, you will have 729 Combinations of 3 Words, the Cube of 9: and if you proceed further,, you will find that each Line doth augment the former 9 times. Poſſibly you may conceive that 6 Words cannot be taken ſo many ſeveral wayes out of 54, the number of Words in the 6 Lines; according to the foregoing Rules, they may be taken 25827165 times, being the number of *Combinations* of 6 Quantities in 54.

The reaſon of the Difference may eaſily be diſcerned, for that you are not to take 2 Words in one Line.

And we further ſubjoin (as a Supplement to, or for the better underſtanding of the 18*th* Section) That the Variations of ſingle Quantities are found by Multiplying the number of Quantities in all the Numbers that are under it, as 6, the Variations of 3 Quantities, is found by Multiplying 3 into 2, into 1. So 120, the Variations of 5 Quantities (or Changes on 5 Bells) is found by Multiplying 5 into 4, in 3, in 2 in, 1.

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*FINIS.*

# ERRATA.

PAge 4. line 23, for *and* 1, read *and* 4. p. 5. l. 14. for 54, r. 5x4. p. 6. l. 27. for 44 × 13, r. 44x43. p. 9. l. 11. for 2, r. 2. p. 10. l. 30, for 2 + 135. r. 21 + 35. p. 11. *l.* 7, Make the top-Figures thus; 1 2 3 4 5 6 7 8. p. 14. l. 10. for 29, r. 20, 8 + 28 + 56 + 70 + 56 + 28 + 5 + 1. *Ibid.* The Table of Variations should have followed *Paragr.* 18. *p.* 13. *Ibid.* to the *9th* Variat. add 0. p. 18. l. 14. for *Sum*, r. *same*. p. 19. l. *ult.* for 3 Chang. r. 4. p. 20. l. 18. *Col.* 1. for 6, r. 4. after 2226, 4, insert 2235, 12. *Ibid.* Cast 14, for 1156. r. 1256. p. 21. *l.* 16. for 8, r. 1. l. 19 for 3, 4, r. 24. 124. for *By* 5, 5. r. *By Seth*, 5. p. 23, l. 24. dele *and*. l. 30, for 4, 5, r. 3, 4, 5. p. 25. *penult.* for 220, r. 120. p. 26. l 10. for *no more*, r. *use more*. l. 30. for $b=3$. r. $b=27$. p. 27. l. 22. for 289, r. 28 *g*. p. 28. l. 11. for $b+6\xi =$ ; r. $b+6\xi = b$. l. 13. for 660, 960. p. 34. *ult.* for 3.42, r. 3x2. p. 48. l. *ult.* for 190, r. 100. p. 49 l. 21. for *the 12th.* r. *the 30th Num. in the 12th.*